PHYSICS

1. A bus moving with constant acceleration covers the distance of 60 m between two points in 5 s. Its speed as it passes the second point is 20 ms^{-1}. At what prior distance from the first point did the bus start from rest ?

(A) 0 (B) 2.5 m (C) 10 m (D) 20 m

1. **(B)** ***Based on One-D Motion Q. 154***

As $v_{avg} = 60 / 5 = 12\ ms^{-1} = (v_1 + v_2) / 2$, $v_1 = 4\ ms^{-1}$.

As $a = (v_2 - v_1) / t = 3.2\ ms^{-2}$, $d = v_1^2 / 2a = 2.5$ m.

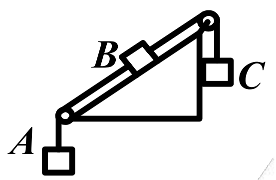
2. A 2 kg block is pressed against a rough vertical wall (coefficient of friction $\mu = 0.6$) by a horizontal force of 40 N perpendicular to the wall. The block's acceleration in ms^{-2} is

(A) 0 (B) 2 (C) 6 (D) 4

2. **(A)** ***Based on Friction Q. 3***

$f_{max} = \mu F_{press} = 24$ N > 20 N = mg. $\therefore a = 0$.

3. The 30º wedge is fixed and there is no friction anywhere. The blocks have masses $m_A = 4$ kg, $m_B = 2$ kg, $m_C = x$ kg. The system is at rest if x =

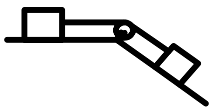


(A) 8 (B) 6 (C) 5 (D) 3

3. **(C)** ***Based on Laws of Motion Q. 51***

$T_{lower} = m_A g$, $T_{upper} = m_C g$, $T_{upper} = T_{lower} + m_B g \sin\theta$ giving $m_C = m_A + m_B \sin\theta$ = 5 kg.

4. Block of mass nm is on the 60^{o} incline while block of mass m is on upper horizontal surface. Friction coefficient for both blocks with surfaces is 0.5. Find the maximum n for the system to be at rest.



(A) $(2\sqrt{3} - 1) / 2$ (B) $2 / (2\sqrt{3} + 1)$ (C) $(2\sqrt{3} + 1) / 2$ (D) $2 / (2\sqrt{3} - 1)$

4. **(D)** ***Based on Friction Q. 35***

When just about to move, $\mu mg = T = nmg (\sin \theta - \mu \cos \theta)$.

$\therefore n = \mu / (\sin \theta - \mu \cos \theta) = 2 / (2\sqrt{3} - 1)$.

5. From points on the horizontal ground that are a distance R apart, two particles are projected simultaneously with velocities $4\sqrt{3}$ ms^{-1} & u at angle 30^{o} & 60^{o} above the horizontal respectively in the same vertical plane towards each other. If the two particles collide after time T, then $(R / T) =$

(A) 12 ms^{-1} (B) 8 ms^{-1} (C) 6 ms^{-1} (D) 4 ms^{-1}

5. **(B)** ***Based on Projectile Motion Q. 89***

To collide, v_{vert} is same i.e. $4\sqrt{3} \sin 30^{o} = u \sin 60^{o}$ or $u = 4$ ms^{-1}.

Also $(R / T) = v_{rel\text{-}horiz} = 4\sqrt{3} \cos 30^{o} + u \cos 60^{o} = 8$ ms^{-1}.

6. Two trains are moving at $v_1 = 10$ ms^{-1} & $v_2 = 20$ ms^{-1} on the same track in opposite directions. When a distance D apart, brakes are applied simultaneously on both trains causing retardations of $r_1 = 2$ ms^{-2} & $r_2 = 1$ ms^{-2} respectively. If a collision is just prevented, then $D =$

(A) 125 m (B) 150 m (C) 225 m (D) 275 m

6. **(C)** ***Based on One-D Motion Q. 169***

$D = (v_1^2 / 2r_1) + (v_2^2 / 2r_2) = 25 + 200 = 225$ m.

7. A horizontal table surface has friction coefficient 0.5. A uniform chain is partly on the table and partly hanging vertically over the edge. Maximum hanging fraction is

(A) 1 / 2 (B) 1 / 3 (C) 2 / 3 (D) 3 / 4

7. **(B)** ***Based on Friction Q. 36***

When just about to slip, $fmg = \mu (1 - f) mg$ or $f = \mu / (1 + \mu) = 1 / 3$.

8. A particle in one-dimensional motion has position x (in m) and time t (in s) related as $x = 5t + 6t^2 - 2t^3$. If its maximum velocity v_M occurs at $t = T$, and the average velocity over the time interval $t = 0$ to $t = T$ is v_{avg}, find $v_{avg} : v_M$.

(A) 2 : 3 (B) 3 : 5 (C) 5 : 9 (D) 9 : 11

8. **(D)** ***Based on One-D Motion Q. 79***

$v = 5 + 12t - 6t^2$, $a = 12 - 12t = 0$ at $T = 1$ s.

$\therefore\ v_M = 5 + 12T - 6T^2 = 11\ \text{ms}^{-1}$, $v_{avg} = (5T + 6T^2 - 2T^3) / T = 9\ \text{ms}^{-1}$.

Ratio = 9 : 11.

9. For a man moving at 1.0 ms^{-1} towards left, rain appears to fall vertically at 2.0 ms^{-1} when wind is blowing at 0.25 ms^{-1} towards right. If wind stops blowing and the man doubles his velocity, find the angle to the vertical at which he should keep his umbrella so that he does not get wet.

(A) $\tan^{-1}(5/8)$ (B) $\tan^{-1}(1/2)$ (C) $\tan^{-1}(\sqrt{5})$ (D) $\tan^{-1}(3/8)$

9. **(D)** ***Based on Projectile Motion Q. 115***

$v_{rain\text{-}vert} = 2\ \text{ms}^{-1}$ and $v_{rain\text{-}left} = (1 + 0.25) = 1.25\ \text{ms}^{-1}$.

$v_{rain\text{-}rel\text{-}horiz} = 2 \times 1 - 1.25 = 0.75\ \text{ms}^{-1}$. $\therefore\ \tan\theta = v_{rain\text{-}rel\text{-}horiz} / v_{rain\text{-}vert} = 3/8$.

10. A 4 kg block is on top of a 5 kg block which is on a smooth horizontal surface. To move the upper block on the lower one, a horizontal 12 N force must be applied to the upper block. The maximum horizontal force that can be applied to the lower block for the blocks to move together is

(A) 30 N (B) 27 N (C) 15 N (D) 12 N

10. **(B)** ***Based on Friction Q. 45***

$a_{max\text{-}common} = 12 / 4 = F / (4 + 5)$ giving $F = 27$ N.

11. A 1 kg block rests on a horizontal table with friction coefficient 0.2. A horizontal force $F = (5 - 2t)$ N acts on it where t is the time in seconds. The frictional force at $t = 2$ s is

(A) 2 N (B) 3 N (C) 1 N (D) 0 N

11. **(C)** ***Based on Friction Q. 32***

$f = min.\ (F, \mu mg) = min.\ (1\ \text{N}, 2\ \text{N}) = 1$ N.

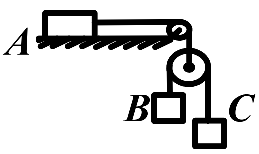
12. A particle's position vector is $\mathbf{r}(t) = (10\,t^2)\,\mathbf{i} + (2 - 20\,t^2)\,\mathbf{j}$ where t is in seconds. The magnitude of its acceleration (in ms^{-2}) at $t = 2$ s is

(A) $20\sqrt{3}$ (B) 20 (C) 60 (D) $20\sqrt{5}$

12. **(D)** ***Based on Projectile Motion Q. 12***

$\mathbf{v}(t) = (20\,t)\,\mathbf{i} - (40\,t)\,\mathbf{j}$, $\mathbf{a}(t) = 20\,\mathbf{i} - 40\,\mathbf{j}$ with $|\,\mathbf{a}\,| = 20\sqrt{5}$ ms^{-2}.

13. 6 kg block A is on a smooth horizontal surface. 0.6 kg block B moves with constant velocity 4 ms^{-1} down. Take $g = 10$ ms^{-2}. Mass of block C is



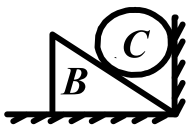
(A) 1 kg (B) 2 kg (C) 3 kg (D) 1.8 kg

13. **(A)** ***Based on Laws of Motion Q. 42***

$T = m_B g = 6$ N. $\therefore a_A = 2T / m_A = 2$ ms^{-2}, $a_C = 2\,a_A = 4$ ms^{-2}.

As $m_C\,(g - a_C) = T, m_C = 1$ kg.

14. All surfaces are smooth. The incline is at 45° and $m_B : m_C = 3 : 2$. The acceleration of B is



(A) $2g / 5$ (B) $3g / 5$ (C) $2g / 3$ (D) $g / 3$

14. **(A)** ***Based on Laws of Motion Q. 55***

$a_C = a_B \tan\theta = a_B$.

From $N\cos\theta = m_C\,(g - a_C)$ and $N\sin\theta = m_B a_B$, $a_B = m_C\,g / (m_B + m_C) = 2g / 5$.

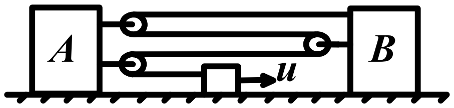
15. A 2 kg block rests on a rough 37º inclined plane with friction coefficient 0.5. Take $g = 10\ ms^{-2}$. The least force applied perpendicular to the plane on the block for it not to slip is

(A) 14 N (B) 12 N (C) 8 N (D) 6 N

15. **(C)** ***Based on Friction Q. 26***

$mg \sin\theta = \mu N = \mu\,(F + mg\cos\theta)$. $\therefore F = (mg / \mu)(\sin\theta - \mu\cos\theta) = 8$ N.

16. If block A moves left at 2 ms^{-1} and block B moves left at 4 ms^{-1}, find u in ms^{-1}.



(A) + 2 (B) + 4 (C) – 2 (D) – 4

16. **(B)** ***Based on Laws of Motion Q. 48***

$-4T \times v_A + 3T \times v_B - T \times u = 0$ gives $u = 4\ ms^{-1}$.

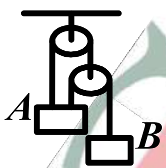
17. A 6 kg block rests on a horizontal table with coefficient of static friction 0.5. Take $g = 10\ ms^{-2}$. The least force to be applied to just move the block is

(A) $12\sqrt{5}$ N (B) $20\sqrt{3}$ N (C) 30 N (D) 60 N

17. **(A)** ***Based on Friction Q. 11***

If F applied at θ above the horizontal, $F\cos\theta = \mu N = \mu\,(mg - F\sin\theta)$ giving $F = \mu mg / (\cos\theta + \mu\sin\theta)$ with $F_{min} = \mu mg / \sqrt{(1 + \mu^2)} = 12\sqrt{5}$ N.

18. If block B is moving down at 6 ms^{-2}, then block A moves up at



(A) 2 ms^{-2} (B) 3 ms^{-2} (C) 6 ms^{-2} (D) 5 ms^{-2}

18. **(A)** ***Based on Laws of Motion Q. 34***

$a_{pulley\text{-}down} = a_A$ and $a_{pulley\text{-}down} = (a_B - a_A) / 2$. $\therefore a_A = a_B / 3 = 2\ ms^{-2}$.
or $3T \cdot a_A - T \cdot a_B = O \Rightarrow a_A = a_B / 3 = 2ms^{-2}$.

19. A particle is projected at 15 ms^{-1} at angle 30º above the horizontal. When its velocity vector is perpendicular to the initial velocity vector, its speed in ms^{-1} is

(A) $5\sqrt{3}$ (B) $15\sqrt{3}$ (C) 30 (D) $10\sqrt{3}$

19. **(B)** ***<u>Based on Projectile Motion Q. 25</u>***

As horizontal velocity unchanged, $15 \cos 30° = v \cos 60°$ or $v = 15\sqrt{3}$ ms^{-1}.

20. A 20 kg block is on a smooth horizontal surface. A force $F = (10\sqrt{2}\, t)$ N where t is in the time in seconds is applied on the block at 45° above the horizontal. The block is at rest at $t = 0$. Take $g = 10$ ms^{-2}. Find the speed with which the block leaves the horizontal surface.

(A) 200 ms^{-1} (B) 120 ms^{-1} (C) 50 ms^{-1} (D) 100 ms^{-1}

20. **(D)** ***<u>Based on Laws of Motion Q. 21</u>***

$mg = F \sin \theta = 10t$ at $t = 20$ s.

As $a = F \cos \theta / m = (t / 2)$, $v = \int_0^{20} (t / 2)\, dt = 20^2 / 4 = 100$ ms^{-1}.

21. An inclined plane makes θ with the horizontal. From a point on it, a projectile is fired with velocity v at right angle to the incline. The range along the incline is $(\mu v^2 / g)$ where μ =

(A) 2 tan θ (B) sec θ (C) 2 tan θ sec θ (D) sin θ

21. **(C)** ***<u>Based on Projectile Motion Q. 60</u>***

$t_{flight} = 2v / (g \cos \theta)$ and $R = ½ (g \sin \theta)\, t_{flight}^2 = (2 \sin \theta / \cos^2 \theta) (v^2 / g)$ giving $\mu = 2 \tan \theta \sec \theta$.

22. A particle starts from the origin at $t = 0$ with initial velocity 5 ms^{-1} and has acceleration at time t (in seconds) as $6t$, the acceleration being in the same direction as the initial velocity. The distance travelled by the particle in 5 s is

(A) 25 m (B) 50 m (C) 150 m (D) 300 m

22. **(C)** ***<u>Based on One-D Motion Q. 124</u>***

As $a = 6t$, $v = v_0 + (6t^2 / 2) = 5 + 3t^2$. ⇒ $s = 5t + t^3 = 150$ m at $t = 5$ s.

23. A 3 kg mass is moving along x-axis at 6 ms^{-1}. A constant force of 12 N is now applied in positive y-direction. Find its speed after 2 s.

(A) 8 ms^{-1} (B) $6\sqrt{5}$ ms^{-1} (C) 10 ms^{-1} (D) $6\sqrt{2}$ ms^{-1}

23. **(C)** ***<u>Based on Laws of Motion Q. 7</u>***

$v_y = Ft / m = 8$ ms^{-1}. As $v_x = 6$ ms^{-1}, $v = 10$ ms^{-1}.

24. A box is on a rough inclined plane of inclination 60º and friction coefficient μ. A force F is applied parallel to the incline upwards on the box. The box's downward motion is prevented by $F = 40$ N and the box's motion up the incline is just started by $F = 80$ N. Then $(1 / \mu)^2 =$

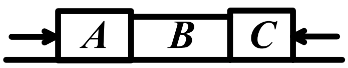
(A) 27 (B) 3 (C) 4 (D) 12

24. **(B)** ***<u>Based on Friction Q. 23</u>***

$80 : 40 = (mg \sin\theta + \mu mg \cos\theta) : (mg \sin\theta - \mu mg \cos\theta) = (\tan\theta + \mu) : (\tan\theta - \mu)$.

$\therefore$ $\mu = (80 - 40) \tan\theta / (80 + 40) = 1 / \sqrt{3}$ or $1 / \mu^2 = 3$.

25. Blocks have masses $m_A = 10$ kg, $m_B = 3$ kg, $m_C = 7$ kg. Forces applied to A & C in the directions indicated are $F_A = 40$ N, $F_C = 20$ N. The horizontal surface is smooth. The ratio of the forces on B by A and C is



(A) 2 : 5 (B) 10 : 9 (C) 9 : 4 (D) 4 : 9

25. **(B)** ***<u>Based on Laws of Motion Q. 23</u>***

$a = (F_A - F_C) / (m_A + m_B + m_C) = 1$ ms^{-2}.

$F_{AB} = FA - m_A a = 30$ N, $F_{BC} = F_{AB} - m_B a = 27$ N. $\therefore$ $F_{AB} : F_{BC} = 10 : 9$.

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26. The number of real solutions of $(x + 2)^{-1} + (x + 7)^{-1} = (x + 4)^{-1} + (x + 5)^{-1}$ is

(A) 0 (B) 1 (C) 2 (D) 3

26. **(B)** ***Based on Basic of Algebra Q. 1***

$(2x + 9)(x^2 + 9x + 20) = (2x + 9)(x^2 + 9x + 14)$ gives $x = -9/2$ i.e. 1 solution only.

27. If $\log_5 3$, $\log_5 (3^x - 30)$, $\log_5 (10 \cdot 3^x + 57)$ are in arithmetic progression, then find the sum of all possible values of x.

(A) 6 (B) 4 (C) 7 (D) 8

27. **(B)** ***Based on Progressions Q. 5***

$(3^x - 30)^2 = 3 \cdot (10 \cdot 3^x + 57)$ or $3^{2x} - 90 \cdot 3^x + 729 = 0$ i.e. $(3^x - 81)(3^x - 9) = 0$.

$\therefore x = 4$ or 2. As $3^x > 30$, $x \neq 2$. $\therefore x = 4$ only.

28. The maximum value of $\sin(395^\circ - A)\cos(385^\circ + A) + \cos(395^\circ - A)\sin(385^\circ + A)$ is

(A) 0 (B) 1 (C) $-\sqrt{3}/2$ (D) $\sqrt{3}/2$

28. **(D)** ***Based on Basic Trigonometry Q. 35***

Expr. $= \sin((395^\circ - A) + (385^\circ + A)) = \sin 780^\circ = \sin 60^\circ = \sqrt{3}/2$.

29. Let p, q, r be distinct reals such that $p^3 + q^3 + r^3 = 3pqr$. Then the roots of $px^2 + 2qx + 4r = 0$ are

(A) real roots (B) lying on either side of 2

(C) both negative (D) non-real

29. **(A)** ***Based on Basic of Algebra Q. 30***

As $(p + q + r)((p - q)^2 + (q - r)^2 + (r - p)^2) = 0$, $p + q + r = 0$. As $f(2) = 0$, 2 is a root i.e. both roots are real.

30. Three numbers, in arithmetic progression, have their sum as 75 and the sum of their squares as 2075. The smallest number among them is

(A) 20 (B) 15 (C) 18 (D) 21

30. **(B)** ***<u>Based on Progressions Q. 30</u>***

If $(25 - d), 25, (25 + d)$; then $2075 = (25 - d)^2 + 25^2 + (25 + d)^2 = 3 \cdot 25^2 + 2d^2$ giving $d = \pm 10$. $\therefore$ Smallest number is $25 - 10 = 15$.

31. For $0 < 3A < \pi$, if $y \sqrt{(1 - \cos 3A)} = x \sqrt{(1 + \cos 3A)}$, then $\cot 3A =$

(A) $(y^2 - x^2) / 2xy$ (B) $(x^2 - y^2) / 2xy$

(C) $(x^2 - y^2) / (y^2 + x^2)$ (D) $(y^2 - x^2) / (y^2 + x^2)$

31. **(A)** ***<u>Based on Basic Trigonometry Q. 92</u>***

$x / y = \sqrt{((1 - \cos 3A) / (1 + \cos 3A))} = \tan 1.5A$.

$\therefore \cot 3A = (1 - \tan^2 1.5A) / (2 \tan 1.5A) = (y^2 - x^2) / 2xy$.

32. Let $\alpha \neq \beta$ but $\alpha^2 = 2\alpha + 1$ and $\beta^2 = 2\beta + 1$. The quadratic equation with roots as $(\alpha + \beta^{-1})$ and $(\beta + \alpha^{-1})$ is

(A) $x^2 - 2x - 1 = 0$ (B) $x^2 = 0$

(C) $x^2 + 2x - 1 = 0$ (D) $x^2 + x - 2 = 0$

32. **(B)** ***<u>Based on Basic of Algebra Q. 81</u>***

α, β are roots of $x^2 - 2x - 1 = 0$ i.e. $\alpha\beta = -1$.

As $(\alpha + \beta^{-1}) = (\alpha\beta + 1) / \beta = 0$, both roots are 0 for $x^2 = 0$.

33. Let α and β be the roots of $375x^2 - 25x - 2 = 0$. Then $\sum_{k=1}^{\infty} (\alpha^k + \beta^k) =$

(A) 21 / 352 (B) 1 / 12 (C) 29 / 398 (D) 725 / 348

33. **(B)** ***<u>Based on Progressions Q. 75</u>***

$S = \alpha (1 - \alpha)^{-1} + \beta (1 - \beta)^{-1} = ((\alpha + \beta) - 2\alpha\beta) / (1 - (\alpha + \beta) + \alpha\beta)$

$= ((25 / 375) + (4 / 375)) / (1 - (25 / 375) - (2 / 375)) = 29 / 348 = 1 / 12$.

34. $(1 - \cot 18^\circ)(1 - \cot 27^\circ) =$

(A) 1 (B) -1 (C) 2 (D) -2

34. **(C)** ***Based on Basic Trigonometry Q. 51***

As $(\cot 18^\circ \cot 27^\circ - 1) / (\cot 18^\circ + \cot 27^\circ) = \cot 45^\circ = 1$,

Expr. $= (1 + \cot 18^\circ \cot 27^\circ) - (\cot 18^\circ \cot 27^\circ - 1) = 2$.

35. If $x^2 - 5x + 7 = 0$ and $x^2 + ax + b = 0$ have a common root, where a & b are real, then $a + b =$

(A) 0 (B) -1 (C) 1 (D) 2

35. **(D)** ***Based on Basic of Algebra Q. 104***

$x^2 - 5x + 7 = 0$ has $D < 0$. Thus both roots are common and $a = -5$, $b = 7$, $a + b = 2$.

36. $\sum_{k=1}^{20} (k / 2^k) =$

(A) $2 - 3 \cdot 2^{-17}$ (B) $1 - 11 \cdot 2^{-20}$ (C) $2 - 11 \cdot 2^{-19}$ (D) $2 - 21 \cdot 2^{-20}$

36. **(C)** ***Based on Progressions Q. 190***

$S - (S / 2) = \left(\sum_{k=1}^{20} (1 / 2^k)\right) - (20 / 2^{21}) = (1 - 2^{-20}) - 10 \cdot 2^{-20}$.

$\therefore S = 2 - 11 \cdot 2^{-19}$.

37. The number of solutions of $| x^2 + 6x + 5 | + 4x + 11 = 0$ is

(A) 0 (B) 2 (C) 3 (D) 4

37. **(B)** ***Based on Basic of Algebra Q. 176***

If $x^2 + 6x + 5 \geq 0$, then $x^2 + 10x + 16 = 0$ giving $x = -8$ or -2 with only $x = -8$ acceptable. If $x^2 + 6x + 5 < 0$, then $x^2 + 2x - 6 = 0$ giving $x = -1 \pm \sqrt{7}$ with only $x = -1 - \sqrt{7}$ acceptable.

38. If $a_1, a_2, a_3, \ldots, a_n$ are in A.P. and $h_1, h_2, h_3, \ldots, h_n$ are in H.P. with $a_3 = h_2 = 8$ and $a_8 = h_7 = 20$, then $a_5 h_{10} =$

(A) 2650 (B) 2560 (C) 3200 (D) 1600

38. **(B)** ***Based on Progressions Q. 168***

$a_5 = a_3 + (2 / 5)(a_8 - a_3) = 12.8$; $h_{10}^{-1} = h_2^{-1} + (8 / 5)(h_7^{-1} - h_2^{-1}) = (1 / 8) - (3 / 25) = 1 / 200$. $\therefore a_5 h_{10} = 12.8 \times 200 = 2560$.

39. $(\cos^2 33^\circ - \cos^2 57^\circ) : (\sin 21^\circ - \cos 21^\circ) =$

(A) $-1/\sqrt{2}$ (B) $1/\sqrt{2}$ (C) $\sqrt{2}$ (D) $-\sqrt{2}$

39. **(A)** ***Based on Basic Trigonometry Q. 77***

$\cos^2 33^\circ - \cos^2 57^\circ = \sin(57^\circ + 33^\circ)\sin(57^\circ - 33^\circ) = \sin 24^\circ = \sin(45^\circ - 21^\circ)$
$= (-1/\sqrt{2})(\sin 21^\circ - \cos 21^\circ)$. $\therefore$ Expr. $= -1/\sqrt{2}$.

40. The complete solution set of all real x such that $\log_{0.3}(\log_3((x^2 + 4x + 3)/(x + 3))) < 0$ is

(A) $(-\infty, 2)$ (B) $(2, \infty)$ (C) Φ (D) $\{2\}$

40. **(B)** ***Based on Basic of Algebra Q. 251***

$\log_3((x^2 + 4x + 3)/(x + 3)) > 1$; $(x^2 + 4x + 3)/(x + 3) > 3$;
$x \neq -3$ and $(x + 1) > 3$. $\therefore x > 2$.

41. For positive numbers m & n, the arithmetic mean is five times the geometric mean. Then $|m + n| : |m - n| =$

(A) $7\sqrt{3} : 12$ (B) $3\sqrt{2} : 4$ (C) $\sqrt{6} : 2$ (D) $5\sqrt{6} : 12$

41. **(D)** ***Based on Progressions Q. 141***

As $(m + n)/2 = 5\sqrt{(mn)}$, $(m - n)^2 = (100 - 4)\,mn = 96\,mn$.
$\therefore$ Ratio $= 10 : \sqrt{96} = 5\sqrt{6} : 12$.

42. If $3y = \pi$, the minimum value of $2\cos^2(x + y) + 2\cos^2(x - y)$ is

(A) 1 (B) $\sqrt{3}$ (C) 2 (D) $2\sqrt{2}$

42. **(A)** ***Based on Basic Trigonometry Q. 148***

Expr. $= 2 + \cos(2x + 2y) + \cos(2x - 2y) = 2 + 2\cos 2x \cos 2y = 2 - \cos 2x \geq 1$.

43. If one root of $x^2 - (k + 1)x + k^2 + k - 8 = 0$ exceeds 2 and the other is smaller than 2, then the complete range of k is

(A) $(-2, 2)$ (B) $(-3, 3)$ (C) $(-2, 3)$ (D) $(-3, 2)$

43. **(C)** ***Based on Basic of Algebra Q. 312***

$a\,f(2) < 0$ if $4 - 2(k + 1) + k^2 + k - 8 < 0$ i.e. $(k - 3)(k + 2) < 0$. $\therefore k \in (-2, 3)$.

44. If $(3^3 - 3)^{-1} + (4^3 - 4)^{-1} + (5^3 - 5)^{-1} + \dots + (101^3 - 101)^{-1} = (N / 101)$, then $N =$

(A) 25 / 3 (B) 143 / 17 (C) 849 / 100 (D) 863 / 102

44. **(B)** ***<u>Based on Progressions Q. 251</u>***

$1 / (k^3 - k) = (1 / 2)\left((k^2 - k)^{-1} - (k^2 + k)^{-1}\right)$. $\therefore S = (1 / 2)\left((3^2 - 3)^{-1} - (101^2 + 101)^{-1}\right)$

and $N = ½ ((101 / 6) - (1 / 102)) = 1716 / 204 = 143 / 17$.

45. If $(k - 1)(\sec x - \tan x) = (k + 1)$; then $(k^2 + 1) \cos x =$

(A) $2k$ (B) $k^2 - 1$ (C) $1 - k^2$ (D) 2

45. **(B)** ***<u>Based on Basic Trigonometry Q. 11</u>***

$(k + 1)(\sec x + \tan x) = (k - 1)$.

$\therefore 2 \sec x = ((k + 1)^2 + (k - 1)^2) / (k^2 - 1) = 2 (k^2 + 1) / (k^2 - 1)$.

$\therefore (k^2 + 1) \cos x = (k^2 - 1)$.

46. Consider the sets {3}, {6, 9, 12}, {15, 18, 21, 24, 27}, **....** The sum of the elements in the 11^{th} set is

(A) 5993 (B) 6893 (C) 6993 (D) 2893

46. **(C)** ***<u>Based on Progressions Q. 219</u>***

Last element of 10^{th} set $= 3 (1 + 3 + 5 + \dots + 19) = 3 \cdot 10^2 = 300$.

$\therefore S_{11} = 303 + 306 + 309 + \dots$ 21 terms $= (21 / 2)(303 + 363) = 6993$.

47. A triangle has angles in arithmetic progression. The ratio of the radian measure of the smallest angle to the degree measure of the mean angle is $\pi : 200$. The smallest angle of the triangle is

(A) 54° (B) 50° (C) 48° (D) 36°

47. **(A)** ***<u>Based on Basic Trigonometry Q. 2</u>***

If $x°$, $60°$, $(120° - x°)$ be angles, then $(\pi x / 180) : 60 = \pi : 200$ or $x = 54°$.

48. $1 + (5 / 6) + (12 / 6^2) + (22 / 6^3) + (35 / 6^4) + (51 / 6^5) + (70 / 6^6) + \dots$ to infinite terms =

(A) 425 / 216 (B) 429 / 216 (C) 288 / 125 (D) 280 / 125

48. **(C)** ***<u>Based on Progressions Q. 207</u>***

$S - (S / 6) = 1 + (4 / 6) + (7 / 6^2) + (10 / 6^3) + \dots$.

$\therefore (5S / 6) - (5S / 36) = 1 + (3 / 6) + (3 / 6^2) + (3 / 6^3) + \dots = 1 + (1 - 6^{-1})^{-1}(3 / 6) = 8 / 5$.

$\therefore S = 288 / 125$.

49. If $7x = \pi$, then $64 \cos 2x \cos 4x \cos 6x =$

(A) 7 (B) 8 (C) -8 (D) -7

49. **(B)** ***Based on Basic Trigonometry Q. 122***

Expr. $= 64 \cos 2x \cos 4x (-\cos x) = -64 \sin 8x / (8 \sin x) = -8 (-\sin x) / (\sin x) = 8$.

50. If $(-\pi / 2) < \theta < 0$ and $\sec \theta = 2$, then $\tan 2\theta =$

(A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $1 / \sqrt{3}$ (D) $-1 / \sqrt{3}$

50. **(A)** ***Based on Basic Trigonometry Q. 95***

$\cos \theta = 1 / 2$, $\theta = -\pi / 3$, $\tan 2\theta = -\tan (2\pi / 3) = \sqrt{3}$.

CHEMISTRY

51. A 25 mL of 0.05 M HCl solution was mixed with 75.00 mL 0.01 M KOH solution. 20 mL of the resulting solution was titrated for neutralization using a standard $Ba(OH)_2$ solution and its 25 mL was required. The molarity of $Ba(OH)_2$ solution is
(A) 0.004 M (B) 0.002 M (C) 0.001 M (D) 0.02 M

51. **(B)** $[(0.5 / 10) \times 20] = M(OH^-) \times 25$. So M of $Ba(OH)_2$ is 0.002

52. A X g sample containing CaO and inert impurity absorbs 224 mL of CO_2 at STP. What volume of a 1.0 M HCl would be required to neutralize $(1/2)X$ g of the same sample?
(A) 5.0 mL (B) 10 mL (C) 20 mL (D) 40 mL

52. **(B)** 0.01 / 2 moles of CaO requirs 0.01 mles of HCl = 10 mL

53. A mixture consists of x moles $K_2C_2O_4$ and y moles of KHC_2O_4. If the mixture is titrated separately against H_2SO_4 and $NaOH$, the moles of acid and bases required respectively to reach the equivalence point would be
(A) $x + (y / 2)$ and y (B) x and $x + (y / 2)$
(C) x and y (D) $(x / 2)$ and y

53. **(A)** 1 mole $K_2C_2O_4$ reacts with 1 mole H2SO4 and 1 mole KHC_2O_4 reacts with 1 / 2 mole

54. An aqueous solution of 6.3 g oxalic acid dihydrate is made upto 250 ml. The volume of 0.1 N $NaOH$ required completely to neutralise 10 ml of this solution is **:**
(A) 40 ml (B) 20 ml (C) 10 ml (D) 4 ml

54. **(A)** $(6.3 / 63) \times 4 \times 10 = 0.1\ x$ VmL

55. Among the following which statement is **NOT** correct ?
(A) HNO_2 can act both as a reducing agent and as an oxidizing agent but HNO_3 acts only as an oxidising agent.
(B) The oxidation number of phosphorus can vary from – 3 to + 5.
(C) The reaction between $NaOH$ and H_2SO_4 is a redox reaction
(D) Oxidation number can have positive, negative, zero or fractional values.

55. **(C)** It is a neutralization reaction

56. Which statement about oxidation number is **FALSE**?
(A) The oxidation number is the number of electron lost (+ve) or gained (–ve) by an atom during the formation of ionic compounds
(B) For covalent compound, the oxidation number is indicated by the charge that an atom of element would have acquired if the substance would have been ionic.
(C) Oxidation number may have integral or fractional values
(D) Oxidation number is same as valency.

56. **(D)** In $Fe(CO)_5$ iron has 5 valency but ON is zero.

57. The process of oxidation **DOES NOT** involve :
(A) addition of O_2 or removal of H_2 from a molecule
(B) addition of a electronegative element or removal of electropositive element
(C) gain of electrons
(D) positive change in oxidation number.

57. **(C)** It is reduction

58. Indicate in which of the following processes the nitrogen is oxidised?
(A) $NH_4^+ \rightarrow N_2$ (B) $NO_3^- \rightarrow NO$ (C) $NO_2 \rightarrow NO_2^-$ (D) $NO_3^- \rightarrow NH_4^+$

58. **(A)** ON of N is increased

59. If the radius of 3rd Bohr's orbit of H atom is 476 pm, then the radius of 4th Bohr's orbit of H atom would be
(A) $476 \times \frac{4}{3}$ pm (B) $476 \times \frac{16}{9}$ pm (C) $476 \times \frac{3}{4}$ pm (D) $476 \times \frac{9}{16}$ pm

59. **(B)** $r_1 \times n^2 = r_n$

60. If the velocity of an electron in first Bohr's orbit of H-atom is v_1, the velocity in the second orbit will be
(A) v_1 (B) $v_1 / 2$ (C) $v_1 / 4$ (D) $2v_1$

60. *(B)* $v = K / n$ K = Constant

61. The dissociation energy of H_2(g) is 430.50 kJ/ mol. If a sample of hydrogen gas is illuminated with light of wavelength 254 nm, the fraction of incident energy which will be converted into kinetic energy is given by
(A) 8.53 % (B) 2.33 % (C) 1.30 % (D) 17.06 %

61. **(A)**

62. Which of the following are isoelectronic :
(I) CH_3^+ (II) H_3O^+ (III) NH_3 (IV) CH_3^-
(A) I, III (B) III, IV (C) I, II (D) II, III, IV

62. **(D)**

63. If the mass attributed to neutron was reduced halved and that attributed to the electron was doubled, the atomic mass of $^{12}C_6$ would be approximately :
(A) Same (B) Doubled
(C) Halved (D) Reduced by 25%

63. **(D)** (12-9) / 12

64. The radii of maximum probability for 3*s*, 3*p* and 3*d* electrons are in the order :
(A) $(r_{\mathbf{max}})_{3d} > (r_{\mathbf{max}})_{3p} > (r_{\mathbf{max}})_{3s}$ (B) $(r_{\mathbf{max}})_{3d} > (r_{\mathbf{max}})_{3s} > (r_{\mathbf{max}})_{3p}$
(C) $(r_{\mathbf{max}})_{3s} > (r_{\mathbf{max}})_{3p} > (r_{\mathbf{max}})_{3d}$ (D) None of these

64. **(C)**

65. The orbital angular momentum of 3p electron is :
(A) $\sqrt{3}\ h$ (B) $\sqrt{6}\ h$ (C) zero (D) $\sqrt{2}\frac{h}{2\pi}$

65. **(D)** Orbital angular momentum $= \sqrt{1(1+1)}\frac{h}{2\pi}$; $l = 1$ for *p*-orbital

66. The ratio of magnetic moments of *Fe* (III) and *Co* (III) is :
(A) $\sqrt{5} : \sqrt{7}$ (B) $\sqrt{35} : \sqrt{15}$ (C) 7 : 3 (D) $\sqrt{24} : \sqrt{15}$

66. **(B)** *Fe*(III) [*Ar*] $3d^5$; unpaired electrons = 5; magnetic moment $= \sqrt{5 \times (5+2)}$ BM

CO(II) – [*Ar*] $3d^7$; unpaired electrons = 3; magnetic moment $= \sqrt{3(3+2)}$ BM

67. How do the energy gaps between successive electron levels in an atom vary from low to high *n* values ?
(A) All energy gaps are the same
(B) The energy gap decreases as *n* increases
(C) The energy gap increases as *n* increases
(D) The energy gap changes unpredictably as *n* increases

67. **(B)**

68. 2.0 g of a crystal of $CaCO_3$ is dissolved in 50 mL water and then mixed with 50 mL of a HCl solution. The resulting solution is boiled to remove all CO_2 and its 10 mL portion required 8.0 mL of a $NaOH$ solution to make the solution neutral. Also 20 mL of original HCl solution is equivalent to 96 mL of $NaOH$ solution. What is the normality of both $NaOH$ and HCl solution.

(A) 1.2 and 0.25 (B) 0.25 and 1.2 (C) 1.4 and 1.2 (D) 0.5 and 1.2

68. **(B)** Let N_1 be normality of HCl and N_2 be the normality of $NaOH$

$\Rightarrow 20\ N_1 = 96\ N_2$ .... (i)

Also, $\frac{2}{50} \times 1000 + 8N_2 \times 10 = 50N_1$

$\Rightarrow 4 + 8N_2 = 5N_1$

$N_2 = 0.25,\ N_1 = 1.2$

69. 1.0 g sample containing KO_2 and some inert impurity is dissolved in excess of aqueous HI solution and finally diluted to 100 mL. The solution is filtered off and 20 mL of filtrate required 15 mL 0.4 M $Na_2S_2O_3$ solution to reduce the liberted iodine. The mass % of KO_2 in the original sample would be

(A) 62 (B) 51 (C) 71 (D) 49

69. **(C)** The balanced redox reaction is :

$2KO_2 + 6HI \longrightarrow 2KOH + 2H_2O + 3I_2$

meq of hypo = 15 × 0.4 = 6 = meq of I_2

$\Rightarrow$ Total I_2 liberated = 15 m mol $\Rightarrow$ m mol of KO_2 in original sample = 10

$\Rightarrow$ m% = $10 \times 10^{-3} \times 71 \times 100 = 71\%$

70. To a 10 mL 1.0 M aqueous solution of Br_2, excess of $NaOH$ is added so that all Br_2 disproportionated to Br^- and BrO_3^-. The resulting solution is free from bromide ion by extraction and excess of OH^- neutralised by acidifying the solution. The resulting solution is just sufficient to react with 1.5 g of a impure CaC_2O_4 sample. The percentage purity of oxalate sample would be

(A) 31.4 (B) 41.2 (C) 25.2 (D) 85.33

70. **(D)** $3Br_2 + 6OH^- \longrightarrow 5Br^- + BrO_3^- + 3H_2O$

$\Rightarrow$ m mol of $BrO_3^- = \frac{10}{3}$

$\Rightarrow$ meq. of $BrO_3^- = \frac{10}{3} \times 6 = 20$ = meq of CaC_2O_4

$\Rightarrow$ mass of $CaC_2O_4 = 20 \times 10^{-3} \times 64 = 1.28$ g

$\Rightarrow$ m% = 85.33

71. An aqueous solution contains 1 mol of $S_2O_3^{2-}$ ions and this reduces 4 mol of Cl_2 molecules. What is the sulphur-containing product of this reaction ?

(A) SO_4^{2-} (B) $S_4O_6^{2-}$ (C) SO_3^{2-} (D) SO_2

71. **(A)** $4Cl_2 + 8e^- \longrightarrow 8Cl^-$

4 moles of Cl_2 gain 8 moles of electrons. Therefore 1 mole of $S_2O_3^{2-}$ have to loose 8 moles of electrons and each S atom has to loose 4 electrons. hence, the oxidation state of S increases by + 4 units, i.e., the final oxidation state is +6.

72. In which of the following arrangements, the order is **NOT** correct according to the property indicated against it :

(A) Increasing size : $Al^{3+} < Mg^{2+} < Na^+ < F^-$
(B) Increasing IE$_1$: $B < C < N < O$
(C) Increasing EA_1 : $I < Br < F < Cl$
(D) Increasing metallic radius : $Li < Na < K < Rb$

72. **(B)**

73. Which one of the following arrangements represents the correct order of electron gain enthalpy (with negative sign) of the given atomic species :

(A) $F < Cl < O < S$ (B) $S < O < Cl < F$
(C) $Cl < F < S < O$ (D) $O < S < F < Cl$

73. **(D)**

74. Lanthanoid contraction is caused due to :

(A) The appreciable shielding on outer electrons by 4*f* electrons from the nuclear charge
(B) The appreciable shielding on outer electrons by 5*d* electrons from the nuclear charge
(C) The same effective nuclear charge from *Ce* to *Lu*
(D) The imperfect shielding on outer electrons by 4*f* electrons from the nuclear charge.

74. **(D)**

75. The charge/size ratio of a cation determines its polarising power. Which one of the following sequences represents the increasing order of the polarising order of the polarising power of the cationic species, K^+, Ca^+, Mg^{2+}, Be^{2+}?

(A) Mg^{2+}, Be^{2+}, K^+, Ca^{2+} (B) Be^{2+}, K^+, Ca^{2+}, Mg^{2+}
(C) K^+, Ca^{2+}, Mg^{2+}, Be^{2+} (D) Ca^{2+}, Mg^{2+}, Be^{2+}, K^+

75. **(C)**