PHYSICS

1. The block shown is on a smooth wedge of angle θ which is fixed on a horizontal rotating circular table. The table rotates with constant angular velocity ω about the vertical axis shown. The block is at distance s from the axis of rotation. The block does not to slip on the wedge if $\omega^2 s$ =

(A) $g \tan \theta$ (B) $2g \tan \theta$ (C) $g \cot \theta$ (D) $2g \cot \theta$

1. **(A)** Balancing forces along incline, $m\omega^2 s \cos \theta = mg \sin \theta$ giving $\omega^2 s = g \tan \theta$.

2. A driver takes 0.25 s to apply the brakes after he sees an obstacle on the road. If he is driving at 20 ms^{-1} and the brakes cause a deceleration of 8 ms^{-2}, the stopping distance after seeing an obstacle is

(A) 25 m (B) 22.5 m (C) 30 m (D) 27.5 m

2. **(C)** $d_1 = u\, t_{react} = 5$ m, $d_2 = u^2 / 2a = 25$ m, $d_{total} = d_1 + d_2 = 30$ m.

3. Car C starts from rest with acceleration 6 ms^{-2} which decreases to zero linearly with time in 10 s. After this, C moves at constant speed. The time needed for C to travel 500 m is

(A) 22 s (B) 20 s (C) 18 s (D) 16 s

3. **(B)** For $0 \le t \le 10$, $a = 0.6\,(10 - t)$, $v = 0.3\,(20t - t^2)$, $s = 0.1\,(30t^2 - t^3)$.
$\therefore\ v_{10} = 30\ ms^{-1}$, $s_{10} = 200$ m. $\therefore\ T = 10 + ((500 - s_{10}) / v_{10}) = 20$ s.

4. A block of mass m is placed on a smooth 45° wedge of mass $2m$. The wedge rests on the horizontal ground with coefficient of friction μ. The least μ for the wedge not to move when the block slides down is

(A) $1 / \sqrt{2}$ (B) $1 / 4$ (C) $1 / 5$ (D) $1 / \sqrt{8}$

4. **(C)** $N_{block} = mg \cos \theta$, $f = N_{block} \sin \theta$, $N_{ground} = 2mg + N_{block} \cos \theta$.
As $f \le \mu N_{ground}$, $\mu_{least} = (\cos \theta \sin \theta) / (2 + \cos^2 \theta) = 1 / 5$.

5. Five bulbs, rated as (100 W, 200 V) are wired in series and connected to a 200 V power supply. The power dissipated by each bulb is

(A) 4 W (B) 20 W (C) 100 W (D) 500 W

5. **(A)** As $R_{eq} = 5$ R, $i_{eq} = i / 5$. As $P \propto i^2$, $P_{each} = 100 / 5^2 = 4$ W.

6. Two stones are thrown simultaneously at $t = 0$ from a height of 15 m with speed 20 ms^{-1} at 30º to the horizontal. They hit the ground at times $t = t_1$ & $t = t_2$ ($> t_1$). Take $g = 10$ ms^{-2}. Then $t_2 : t_1 =$

(A) 3 : 1 (B) 2 : 1 (C) 3 : 2 (D) 4 : 3

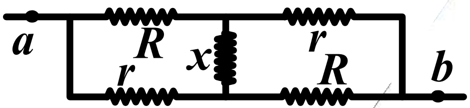
6. **(A)** $u_v = \pm 20 \sin 30º = \pm 10$ ms^{-1}. As $5t_1^2 + 10t_1 = 15$ & $5t_2^2 - 10t_2 = 15$, $t_1 = 1$ s & $t_2 = 3$ s.

7. A man walking down a 30º incline with speed $\sqrt{(12)}$ ms^{-1} has to keep his umbrella vertical to protect himself from rain. The actual speed of rain is 5 ms^{-1}. If he stops walking, he needs to hold the umbrella at angle θ with the vertical to not get drenched. Then tan θ =

(A) 3 / 4 (B) 4 / 3 (C) 1 / 3 (D) 1 / √3

7. **(A)** $v_{rain\text{-}horiz} = \sqrt{12} \cos 30º = 3$ ms^{-1} $= v_{rain} \sin \theta$. $\therefore \sin \theta = 3/5$, $\tan \theta = 3/4$.

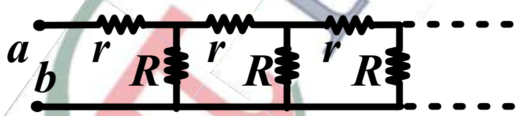
8. If $R = 10\ \Omega$, $r = x = 5\ \Omega$, the equivalent resistance (to the closest Ω) between the points a & b is



(A) 6 Ω (B) 7 Ω (C) 8 Ω (D) 9 Ω

8. **(B)** $i_x = i_r - i_R$, $i_R R = i_r r + i_x x$ giving $i_r = 1.5\ i_R$, $i_x = 0.5\ i_R$.
$\therefore R_{eq} = (i_R R + i_r r) / (i_R + i_r) = 17.5 / 2.5 = 7\ \Omega$.

9. For the infinite ladder of resistors, a voltage is applied between points a & b. If the voltage is halved after each section, $(R : r) =$



(A) 1 (B) 2 (C) 3 (D) 4

9. **(B)** For voltage to halve, $R_{eq} = 2r$. $\therefore R \,//\, 2r = r$ or $R = 2r$.

10. An object thrown vertically up from the ground is 25 m above the ground at two instants that are 4 s apart. Take $g = 10\ ms^{-2}$. The time of flight of the object is

(A) 8 s (B) $3\sqrt{6}$ s (C) $5\sqrt{2}$ s (D) 6 s

10. **(D)** $H - 25 = ½\, g\, (4 / 2)^2 = 20.$ $\therefore H = 45$ m. $\therefore T = 2\sqrt{(2H / g)} = 6$ s.

11. A 80 Ω galvanometer has full scale deflection 20 mV. It is transformed to a voltmeter with full scale deflection 5.0 V by using a resistor R =

(A) 12.20 kΩ (B) 20.54 kΩ (C) 19.92 kΩ (D) 24.40 kΩ

11. **(C)** $R = ((V_V / V_G) - 1)\, R_G = (5000 / 20) \cdot 80 - 80 = 20000 - 80 = 19920\ \Omega = 19.92$ kΩ.

12. A block released from rest from the top of a smooth inclined plane of inclination b has a speed v at the bottom. The same block released from the top of a rough inclined plane of the same inclination b has a speed kv at the bottom. Then the coefficient of friction μ =

(A) $(1 - k^2) \tan b$ (B) $(1 - k^2)^{½} \cot b$
(C) $(1 - k^2)^{½} \tan b$ (D) $(1 - k^2) \cot b$

12. **(A)** $k^2 = (g \sin b - \mu g \cos b) : (g \sin b) = 1 - \mu \cot b$ or $\mu = (1 - k^2) \tan b$.

13. Particles K & L rotate about an axis starting from rest at time $t = 0$. K has constant angular acceleration while B has constant angular velocity. K completes its first revolution in time $\sqrt{\pi}$ seconds & B needs 4π seconds to complete first half-revolution. At $t = 5$ seconds, the ratio of the angular velocity of K to that of L is

(A) 4 : 1 (B) 20 : 1 (C) 80 : 1 (D) 5 : 1

13. **(C)** $\alpha_K = 2\theta_K / t_K^2 = 4\pi / \pi = 4;\ \omega_L = \theta_L / t_L = \pi / 4\pi = 1 / 4.$
$\therefore \omega_K : \omega_L = \alpha_K t : \omega_L = 4 \times 5 : (1 / 4) = 80 : 1.$

14. The block c is being pulled right with acceleration a_c. Due to this, a & b have accelerations a_a & a_b to the left. Then a_c =

(A) $2a_a + a_b$ (B) $2a_a - a_b$ (C) $0.5a_a - a_b$ (D) $0.5a_a + a_b$

14. **(A)** $2T \cdot a_a + T \cdot a_b - T \cdot a_c = 0$ giving $a_c = 2a_a + a_b$.

15. A point moves along a circle of radius 20 cm with constant tangential acceleration 5 cm s^{-2} starting from rest at $t = 0$. Its normal acceleration and tangential acceleration are equal in magnitude at time t =

(A) 1 s (B) 2 s (C) 3 s (D) 4 s

15. **(B)** $a_T = a_N = v^2 / R = (a_T\, t)^2 / R$ gives $t = \sqrt{(R / a_T)} = 2$ s.

16. The displacement s of an object moving along a straight line is directly proportional to the cube of the time t of motion. Its acceleration a is directly proportional to

(A) $s^{1/3}$ (B) $s^{1/2}$ (C) s (D) s^2

16. **(A)** $s = bt^3$, $v = 3bt^2$, $a = 6bt \propto s^{1/3}$.

17. For a simple pendulum, the bob is released with the string taut and horizontal. The ratio of the acceleration magnitudes when the string is vertical to that when the string is horizontal is

(A) 2 (B) 1 / 2 (C) 1 (D) 2 / 3

17. **(A)** $a_{horiz} = g$, $a_{vert} = v^2 / L = (2gL) / L = 2g$. $\therefore a_{vert} : a_{horiz} = 2$.

18. A balloon starts rising from the earth's surface and has a constant vertical component of velocity as u. Due to wind, it has a horizontal velocity component $v_x = ky^2$, where k is a constant and y is the height. Assuming it started from the origin, the equation of its trajectory is

(A) $3ux = ky^3$ (B) $2ux = ky^2$ (C) $uy = kx^2$ (D) $u^2x^2 = ky$

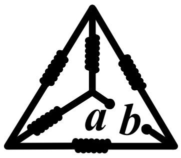
18. **(A)** $y = ut$, $dx / dt = v_x = k\, u^2t^2$, $x = (ku^2 / 3)\, t^3 = (k / 3u)\, y^3$ or $3ux = ky^3$.

19. A uniform chain of length L and mass m is on a smooth table with one-fourth length hanging over the edge. The work done to pull the whole chain back onto the table is

(A) $mgL / 4$ (B) $mgL / 8$ (C) $mgL / 16$ (D) $mgL / 32$

19. **(D)** $\Delta h_{cm\text{-}hang} = \frac{1}{2} L_{overhang} = L / 8$, $m_{hang} = m / 4$, $W = m_{hang}\, g\, \Delta h_{cm\text{-}hang} = mgL / 32$.

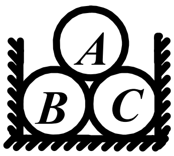
20. In the network shown, each of the five resistances is 2 Ω. The equivalent resistance between points a and b is



(A) 3 Ω (B) 4 Ω (C) 1 Ω (D) 2 Ω

20. **(D)** Two terminals, not adjacent to b, are equipotential and the resistance connecting them can be removed. Then $R_{ab} = (2 + 2) // (2 + 2) = 2\ \Omega$.

21. The three identical cylinders have mass m each. All surfaces are smooth. There is no contact force between B & C. Contact force between left wall & B is



(A) $mg / \sqrt{12}$ (B) $mg / \sqrt{3}$ (C) $mg / 2$ (D) $mg / 3$

21. **(A)** For A : $mg = 2\,N_{vert} = 2\,N_{AB} \sin 60^\circ$ giving $N_{AB} = mg / \sqrt{3}$.
For B : $N_{wall\text{-}B} = N_{AB\text{-}horiz} = N_{AB} \cos 60^\circ = mg / 2\sqrt{3} = mg / \sqrt{12}$.

22. The block of mass m is on a plank of mass M which is on the smooth horizontal floor. The friction coefficient between the block & plank is μ. If the block be given a velocity to the right, the relative acceleration of the block and plank is

(A) $(\mu g / m)(M - m)$ (B) $(\mu g / m)(M + m)$
(C) $(\mu g / M)(M - m)$ (D) $(\mu g / M)(M + m)$

22. **(D)** $f = \mu mg$, $a_{block\text{-}left} = f / m = \mu g$, $a_{plank\text{-}right} = f / M = \mu mg / M$,
$a_{rel} = a_{block\text{-}left} + a_{plank\text{-}right} = (\mu g / M)(M + m)$.

23. A particle starts almost from rest from the origin and has acceleration a related to displacement s as $a = k^2 s$, k being a positive constant. Its velocity v is related to s as

(A) $v = ks$ (B) $v^2 = 2ks$ (C) $2v = k^2 s$ (D) $v^2 = ks^2$

23. **(A)** $v\,dv / ds = a = k^2 s$ or $v\,dv = k^2 s\,ds$. $\therefore v = ks$.

24. A swimmer swims in still water with speed 2.5 kmph. He enters a 200 m wide river, having flow speed of 2 kmph, at point A and proceeds to swim at angle $(90^o + 37^o)$ with the river flow direction. Point B is located directly across A on the other bank. The swimmer lands on the other bank at point C, from which he walks to B with speed 3 kmph. The total time in minutes in which he reaches from A to B is

(A) 10 (B) 9 (C) 8 (D) 7

24. **(D)** $v_{AB} = 2.5 \cos 37^o = 2$ kmph, $v_{BC} = 2 - 2.5 \sin 37^o = 0.5$ kmph. $t_{swim} = 0.1$ hour $= 6$ min,
$d_{CB} = v_{BC}\, t_{swim} = 0.05$ km, $t_{walk} = d_{CB} / v_{walk} = (1/60)$ hr $= 1$ min.
$\therefore t_{total} = 7$ min.

25. Two stones are projected with the same speed of 25 ms^{-1} from the same point and have the same range of 60 m. Find the difference in their time of flight. Note that if $\sin x = 0.6$, then $\sin 2x = 0.96$.

(A) 2 s (B) 1 s (C) $\sqrt{2}$ s (D) $\sqrt{3}$ s

25. **(B)** $\sin 2\theta = gR / u^2 = 600 / 625 = 0.96$. $\therefore \sin\theta = 0.6$ or 0.8.
As $t = (2u / g) \sin\theta$, $t = 3$ s or 4 s. $\therefore \Delta t = 1$ s.

MATHEMATICS

26. $ln\ a$, $ln\ b$, $ln\ c$ are in arithmetic progression and $ln\ a - ln\ 2b$, $ln\ 2b - ln\ 3c$, $ln\ 3c - ln\ a$ are in arithmetic progression.
(A) $2a, b, c$ are in arithmetic progression
(B) $a, 2b, 3c$ are in geometric progression
(C) $a, 2b, 3c$ are in harmonic progression
(D) $c, b, 2a - b - c$ are in arithmetic progression

26. **(D)** $b^2 = ac$ & $(2b / 3c)^2 = (3c / 2b)$. $\Rightarrow a : b : c = 9 : 6 : 4$.
$\therefore$ $c, b, 2a - b - c$ are in arithmetic progression.

27. If p, q, r are non-zero, real numbers such that $p^2 + q^2 + r^2 = pq + qr + rp$, then the roots of $px^2 + qx + r = 0$ are
(A) real & distinct
(B) real & equal
(C) non-real & distinct
(D) non-real & equal

27. **(C)** As $(p - q)^2 + (q - r)^2 + (r - p)^2 = 0$, $p = q = r$. $\therefore x^2 + x + 1 = 0$
which has non-real, distinct roots.

28. $x^3 - 12x + 16 = 0$ has roots p, q, r. The equation with roots $2(q + r)^2 / p^3$, $2(r + p)^2 / q^3$, $2(p + q)^2 / r^3$ is
(A) $2x^3 - 3x^2 + 1 = 0$
(B) $2x^3 - 3x + 1 = 0$
(C) $2x^3 + 3x - 1 = 0$
(D) $2x^3 + 3x^2 - 1 = 0$

28. **(A)** As $p + q + r = 0$, $2(q + r)^2 / p^3 = 2(-p)^2 / p^3 = 2 / p$.
Equation is $(2 / x)^3 - 12\,(2 / x) + 16 = 0$ or $2x^3 - 3x^2 + 1 = 0$.

29. $1^{-1} + (1 + 2)^{-1} + (1 + 2 + 3)^{-1} + \ldots + (1 + 2 + \ldots + n)^{-1} =$
(A) $2n / (n + 1)$ (B) $n / (n + 1)$ (C) $n / (2n + 2)$ (D) $n^2 / (2n - 2)$

29. **(A)** $T_k = 2 / (k\,(k + 1)) = 2\,(k^{-1} - (k + 1)^{-1})$. $\therefore S_n = 2(1^{-1} - (n + 1)^{-1}) = 2n / (n + 1)$.

30. Both roots of $x^2 - 2kx + k^2 - 9 = 0$ are less than $\sqrt{26}$. The greatest integral value of k is
(A) 2 (B) 5 (C) 10 (D) 26

30. **(A)** $(x - k)^2 = 9$. $\therefore x = k \pm 3$. As $k + 3 < \sqrt{26}$, $k_{max} = 2$.

31. The number of integers x satisfying $| x^2 - 2x | - 8 \le 0$ is

(A) 3 (B) 5 (C) 7 (D) 9

31. **(C)** $| (x-1)^2 - 1 | \le 8$, $(x-1)^2 \le 9$. $\therefore$ $(x-1) = 0, \pm 1, \pm 2, \pm 3$ i.e. 7 solutions.

32. Let $x = \tan(7\pi / 6)$, $y = \tan(9\pi / 4)$, $z = \tan(10\pi / 3)$.

(A) $2y = x + z$ (B) $y^2 = xz$ (C) $2xz = xy + yz$ (D) $2y^2 = x^2 + z^2$

32. **(B)** $x = \tan(\pi + (\pi / 6)) = 1 / \sqrt{3}$, $y = \tan(2\pi + (\pi / 4)) = 1$, $z = \tan(3\pi + (\pi / 3)) = \sqrt{3}$.
$\Rightarrow y^2 = xz$.

33. The numbers of integers x satisfying $| x | < 7$ and $(x^2 + 3x + 4) / (1 - x) > 1$ is

(A) 3 (B) 4 (C) 6 (D) 7

33. **(B)** $(x^2 + 4x + 3) / (1 - x) > 0$. $(x + 3)(x + 1)(x - 1) < 0$. $\therefore$ $x < -3$ or $-1 < x < 1$.
$\therefore$ $x = -6, -5, -4, 0$ i.e. 4 values.

34. In a geometric progression with all terms positive, any term is equal to the sum of the next 2 terms. The common ratio can be

(A) $(\sqrt{3} - 1) / 2$ (B) $(\sqrt{3} + 1) / 4$ (C) $(\sqrt{5} - 1) / 2$ (D) $(\sqrt{5} + 1) / 4$

34. **(C)** $1 = r + r^2 \Rightarrow r = ½(-1 \pm \sqrt{5})$. As $r > 0$, $r = (\sqrt{5} - 1) / 2$.

35. The roots of $x^2 + 3x + 2 = 0$ are also the roots of $x^4 - px^2 + q = 0$. The largest root of $x^4 - px^2 + q = 0$ is

(A) 2 (B) 3 (C) 4 (D) 5

35. **(A)** $-1, -2$ satisfy $x^4 - px^2 + q = 0$. $\therefore$ $p = 1 + q$ & $4p = 16 + q$.
$\therefore$ $p = 5, q = 4$. $\therefore$ $x^4 - 5x^2 + 4 = 0$ or $(x^2 - 1)(x^2 - 4) = 0$ i.e. $x = \pm 1, \pm 2$.
$\therefore$ Largest root is 2.

36. The smallest positive solution of $2 \sin x + 1 = 0 = \sqrt{3} \tan x - 1$ is

(A) $2\pi / 3$ (B) $5\pi / 6$ (C) $7\pi / 6$ (D) $4\pi / 3$

36. **(C)** $\sin x = -1 / 2$, $\tan x = 1 / \sqrt{3}$. So smallest $x = 7\pi / 6$.

37. $8(\sin 163^\circ \sin 103^\circ + \sin 73^\circ \sin 167^\circ)^2 =$

(A) 2 (B) 4 (C) 6 (D) $4 - \sqrt{6}$

37. **(A)** $\sin 163^\circ \sin 103^\circ + \sin 73^\circ \sin 167^\circ = \sin 17^\circ \cos 13^\circ + \cos 17^\circ \sin 13^\circ = \sin 30^\circ$
$= 1 / 2$. $\therefore$ Expr. = 2.

38. $\sin^6 x + \cos^6 x = A + B \cos 4x$ is an identity. Then $A - B =$

(A) 31 / 24 (B) 1 / 4 (C) 1 / 2 (D) 1

38. **(B)** $\sin^6 x + \cos^6 x = (\sin^2 x + \cos^2 x)^3 - 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)$

$= 1 - (3 / 4) \sin^2 2x = 1 - (3 / 8) (1 - \cos 4x) = (5 / 8) + (3 / 8) \cos 4x$.

$\therefore A - B = (5 / 8) - (3 / 8) = 1 / 4$.

39. The number of values of x in $[0, 4\pi]$ satisfying $5 \tan^2 x - \sec^2 x = 11$ is

(A) 2 (B) 4 (C) 8 (D) 12

39. **(C)** $5 \tan^2 x - (1 + \tan^2 x) = 11$. $\therefore \tan x = \pm \sqrt{3}$. $\therefore$ 1 value in every interval of $(\pi / 2)$ giving total 8 values.

40. If $3 + ((3 + d) / 4) + ((3 + 2d) / 4^2) + \ldots +$ to infinite terms $= 12$, then $d =$

(A) 9 (B) 18 (C) 24 (D) 32

40. **(B)** $S - ¼ S = 3 + (d / 4) + (d / 4^2) + (d / 4^3) + \ldots$. $\therefore d = 9 (S - 4) / 4 = 18$.

41. For $0 \le x \le 4\pi$, number of solutions of $\sin 2x + \cos 2x = \sin x + \cos x$ is

(A) 4 (B) 8 (C) 9 (D) 6

41. **(C)** $\sin (2x + ¼\pi) = \sin (x + ¼\pi)$. $\Rightarrow \sin ½x \cos (1.5x + ¼\pi) = 0$.

$\therefore x = 2n\pi$ or $x = (4k + 1) \pi / 6$. $\therefore n = 0, 1, 2$ or $k = 0, 1, 2, \ldots, 5$ i.e.

$3 + 6 = 9$ solutions.

42. If $\cot A \operatorname{cosec} B + \operatorname{cosec} A \cot B = 25$, then $(\cot A \cot B + \operatorname{cosec} A \operatorname{cosec} B)^2 =$

(A) 650 (B) 626 (C) 624 (D) 600

42. **(B)** $(\operatorname{cosec} A \operatorname{cosec} B + \cot A \cot B)^2 - (\cot A \operatorname{cosec} B + \operatorname{cosec} A \cot B)^2$

$= \operatorname{cosec}^2 B - \cot^2 B = 1 \Rightarrow$ Expr $= 1 + 625 = 626$.

43. For $2 - \cos x + \sin^2 x$, the ratio of the greatest value to the least value is

(A) 3 : 1 (B) 17 : 6 (C) 13 : 4 (D) 7 : 2

43. **(C)** $p = 3.25 - (\cos x + 0.5)^2$, $p_{max} = 3.25$, $p_{min} = 3.25 - 1.5^2 = 1$.

$\therefore n = 3.25 = 13 : 4$.

44. If $\sin\theta$, $\cos\theta$, $\tan\theta$ are in geometric progression, then $\cos^9\theta + \cos^6\theta + 3\cos^5\theta =$

(A) -1 (B) 0 (C) 1 (D) 2

44. **(C)** $\cos^2\theta = \sin\theta\tan\theta \Rightarrow \cos^3\theta = 1 - \cos^2\theta \Rightarrow \cos^3\theta + \cos^2\theta = 1$. Cubing both sides, $\cos^9\theta + \cos^6\theta + 3\cos^5\theta = 1$.

45. $(1^3 / 1) + ((1^3 + 2^3) / (1 + 3)) + ((1^3 + 2^3 + 3^3) / (1 + 3 + 5)) + \dots$ to 16 terms =

(A) 428 (B) 446 (C) 464 (D) 482

45. **(B)** $T_k = (1^3 + 2^3 + \dots + k^3) / (1 + 3 + \dots + (2k - 1)) = \frac{1}{4}(k + 1)^2$.
$\therefore 4S_{16} + 1 = (1^2 + 2^2 + \dots + 17^2) = 17 \cdot 18 \cdot 35 / 6 = 1785. \therefore S_{16} = 446$.

46. The number of solutions of $\sqrt{3}\cos\theta + \sin\theta = 1$, $-2\pi < \theta < 2\pi$ is

(A) 2 (B) 3 (C) 4 (D) 6

46. **(C)** $2\cos(\theta - (\pi / 6)) = 1$. $\therefore \theta - (\pi / 6) = 2n\pi \pm (\pi / 3)$.
$\therefore \theta = -3\pi / 2, -\pi / 6, \pi / 2, 11\pi / 6$ i.e. 4 solutions.

47. The arithmetic mean of two positive numbers is three times their geometric mean and the sum of their squares is 34. The larger number is

(A) $2\sqrt{3} + \sqrt{5}$ (B) $3 + 2\sqrt{2}$ (C) $\sqrt{10} + \sqrt{7}$ (D) 5

47. **(B)** $a + b = 6(ab)^{1/2}. \Rightarrow a^2 + b^2 = 34ab. \Rightarrow ab = 1 \Rightarrow a - b = \sqrt{32}. \Rightarrow a = 3 + 2\sqrt{2}$.

48. Three distinct numbers x, y, z form a geometric progression and $x + y, y + z, z + x$ form an arithmetic progression. The common ratio of the geometric progression is

(A) -1 (B) 1 (C) 2 (D) -2

48. **(D)** $2x = y + z. \Rightarrow r^2 + r = 2. \Rightarrow r = 1$ (not distinct) or $r = -2$.

49. The sum of all the solutions of $[x] + 2\{x\} = 12.4$, where $[\cdot]$ & $\{\cdot\}$ are integer part & fractional part respectively, is

(A) 12.2 (B) 16.4 (C) 19.8 (D) 23.9

49. **(D)** $\{x\} = 0.2$ or 0.7 giving $[x] = 12$ or 11. $\therefore x = 12.2$ or 11.7.

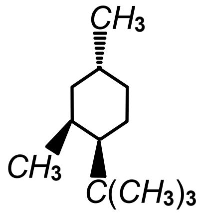
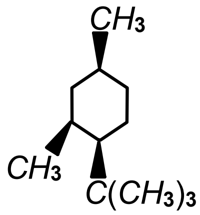
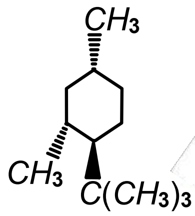
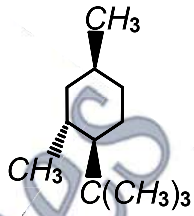
50. If $\alpha + \beta - \gamma = \pi$, then $\sin^2\alpha + \sin^2\beta - \sin^2\gamma =$

(A) $2\sin\alpha\sin\beta\cos\gamma$ (B) $2\cos\alpha\cos\beta\cos\gamma$
(C) $2\sin\alpha\sin\beta\sin\gamma$ (D) $2\cos\alpha\cos\beta\sin\gamma$

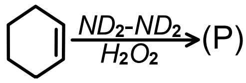
50. **(A)** Expr. $= \sin^2\alpha + \sin(\beta + \gamma)\sin(\beta - \gamma) = \sin\alpha(\sin(\beta - \gamma) + \sin(\beta + \gamma))$
$= 2\sin\alpha\sin\beta\cos\gamma$.

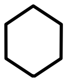
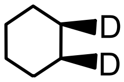
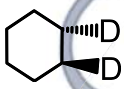
CHEMISTRY

51. Which one of the following has the lowest heat of combustion ?

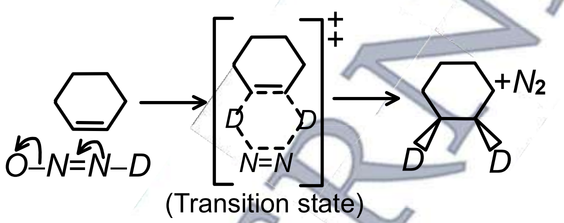
(A) 
(B) 
(C) 
(D) 

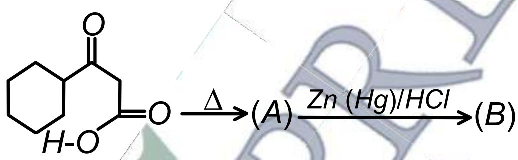
51. **(C)** Is most stable because of less steric repulsion
$\therefore$ Least heat of combustion.

52.  Product (P) is **:**

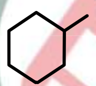
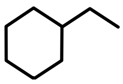
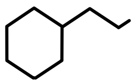
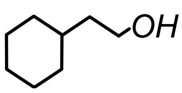
(A) 
(B) 
(C) 
(D) Both (B) and (C)

52. **(B)** Reduction of alkane by pericylic mechanism.
(1) $H_2O_2 + ND_2 - ND_2 \longrightarrow D - N = N - D$

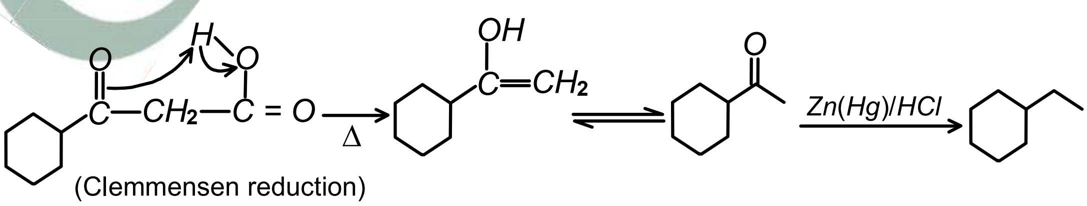
(2) 

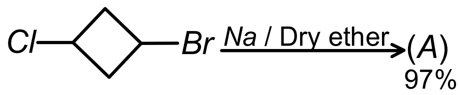
53. 

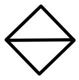
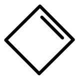
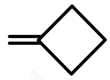
In the above reaction, product (B) is

(A) 
(B) 
(C) 
(D) 

53. **(B)** Reactant in β-keto acid easily undergo decarboxylation.

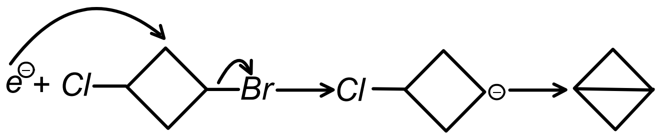


54.  Product (A) of given reaction is **:**

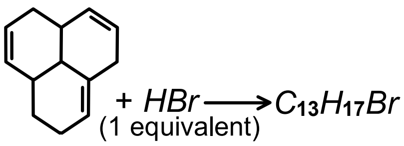
(A)  (B)  (C)  (D) 

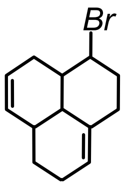
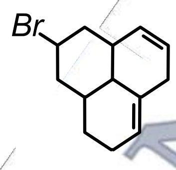
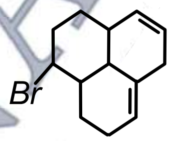
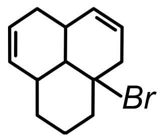
54. **(B)** It is an example of intramolecular Wurtz reaction.

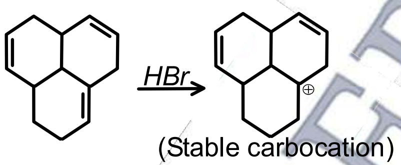
$Na \rightarrow Na^{+} + e^{\ominus}$



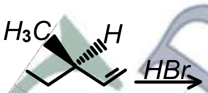
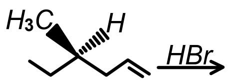
55. Which of the following bromides is the major product of the reaction shown below, assuming that there is no carbocation rearrangement ?

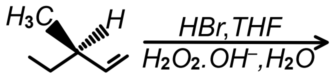


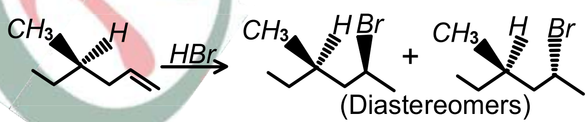
(A)  (B)  (C)  (D) 

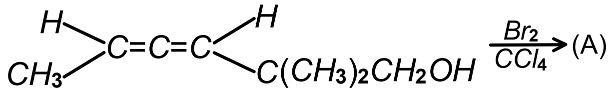
55. **(D)** 

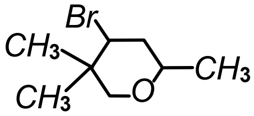
56. Which of the following reactions results in the formation of a pair of diastereomers ?

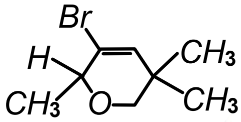
(A)  (B) 

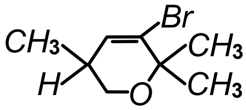
(C)  (D) 

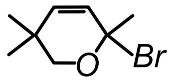
56. **(B)** 

57.  . Product (A) in above reaction is :

(A) 

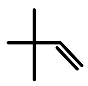
(B) 

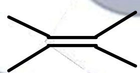
(C) 

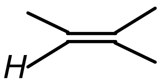
(D) 

57. **(D)** Intramolecular nucleophilic attack.

58. $CH_3-\underset{CH_3}{\overset{CH_3}{|}}\!\!\!CH + H_2C=CH_2 \xrightarrow[2\text{-}5^\circ C]{HF} ;(A)$ is

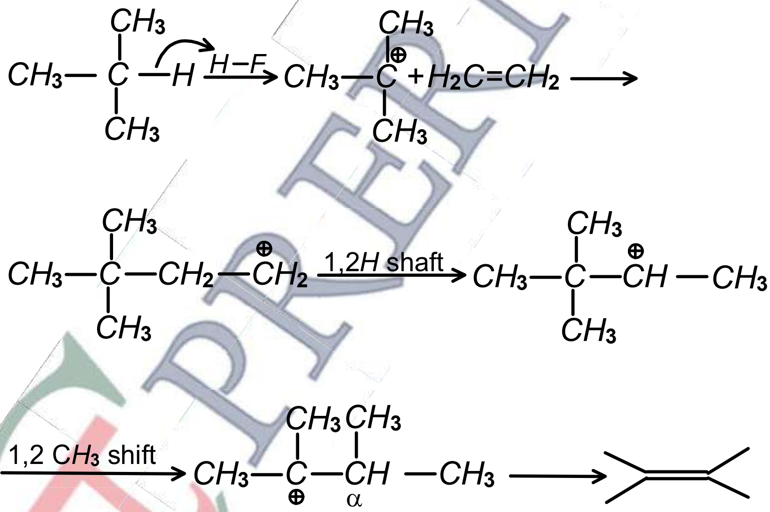
(A) 

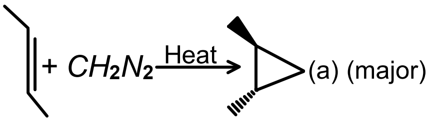
(B) 

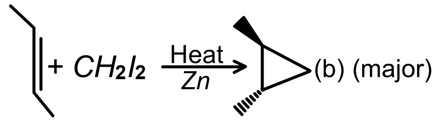
(C) 

(D) $CH_3-\underset{CH_3}{\underset{|}{CH}}-CH_2-CH=CH_2$

58. **(B)**



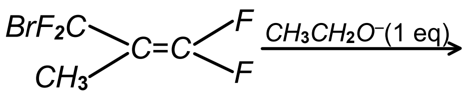
59. Case A 

Case B 

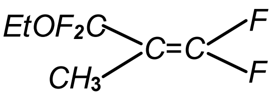
In the above reaction difference in reagents results in the different product mixture. Find relation between a and b.

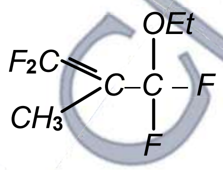
(A) a = b (B) a > b (C) b > a (D) Cannot predict

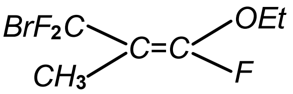
59. **(C)** Zinc stabilised the transition state.

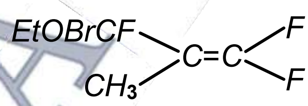
60. 

'*X*' (Major product). Product '*X*' is

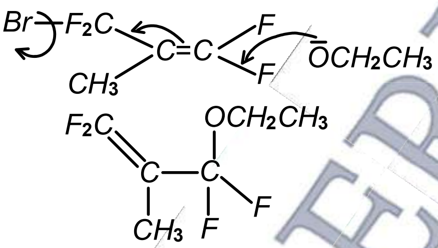
(A) 

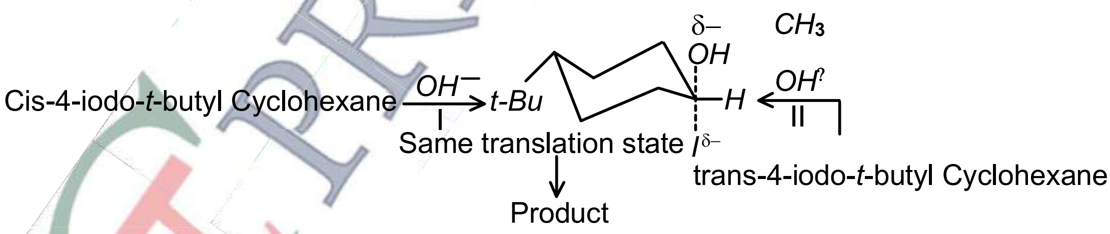
(B) 

(C) 

(D) 

60. **(B)** *Br* is better leaving group.

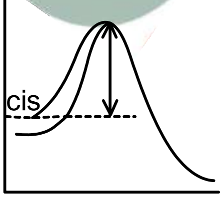


61. 

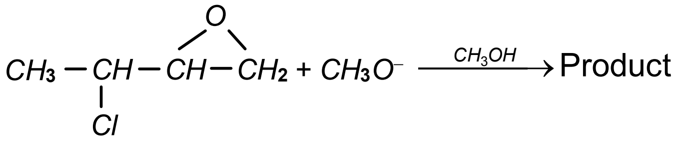
Hence,

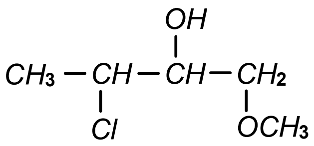
(A) $r_1 = r_2$ (B) $r_1 > r_2$ (C) $r_1 < r_2$ (D) can't be predicted

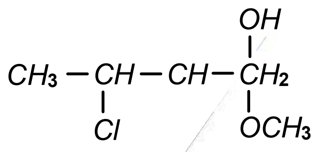
61. **(B)** Same transition state but substrates cis and trans differ in ground state energy.

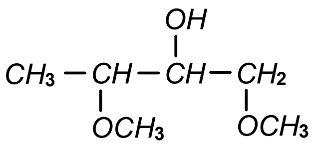


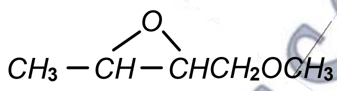
cis will have lower activation energy, thus faster rate of reaction.

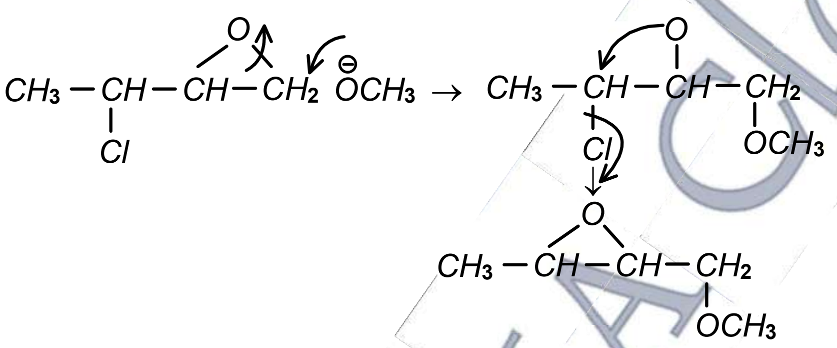
62.  . The predominant product is

(A) 

(B) 

(C) 

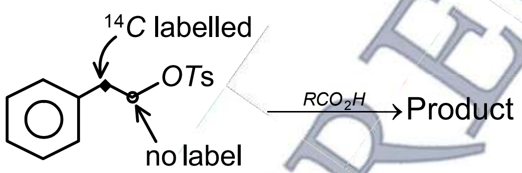
(D) 

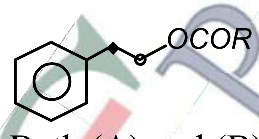
62. **(D)** 

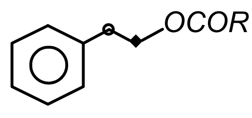
63. Among the following alkyl halides, one with lowest boiling point is

(A) *t*-butyl chloride
(B) *n*-butyl chloride
(C) *n*-butyl iodide
(D) *t*-butyl iodide

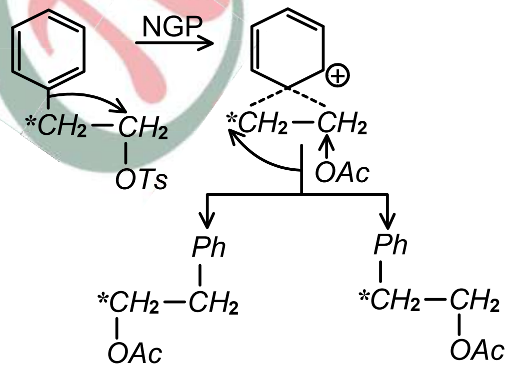
63. **(A)**

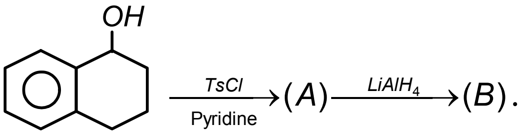
64.  . Product of this reaction is

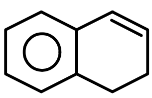
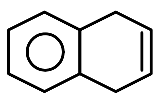
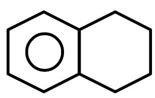
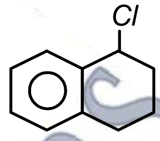
(A) 

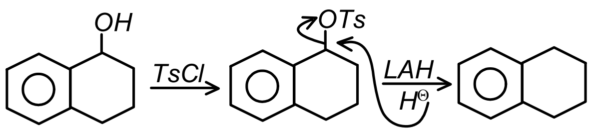
(B) 

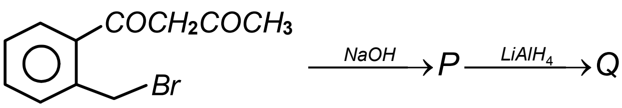
(C) Both (A) and (B)
(D) None of these

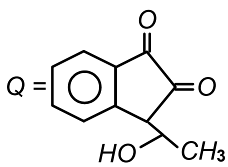
64. **(C)** 

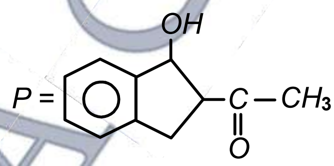
65.  Product (B) of the above reaction is

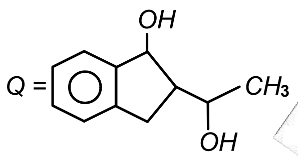
(A)  (B)  (C)  (D) 

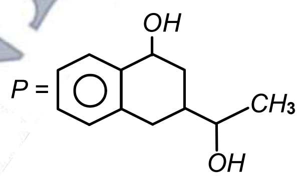
65. **(C)** 

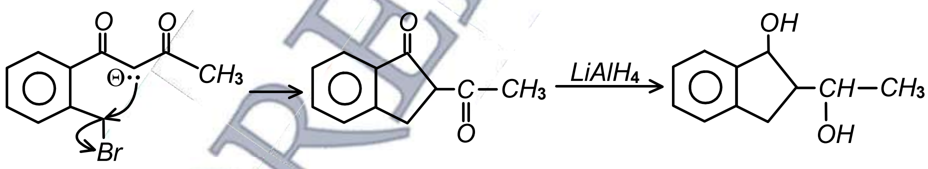
66.  Which of the following statement is correct?

(A) 

(B) 

(C) 

(D) 

66. **(C)** 

67. A first order reaction is carried out with an initial concentration of 10 mole per litre and 80% of the reactant change into the product in 10 sec. Now if the same reaction is carried out with an initial concentration of 5 mol per litre the percentage of the reactant changing to the product in 10 sec is:

(A) 20 (B) 80 (C) 60 (D) 100

67. **(B)** Time required to complete a definite fraction is independent of initial concentration for 1st order reaction.

68. The rate constant is numerically the same for three reactions of first, second and third order respectively. Which one is true for rate of three reactions, if concentration of reactants is greater than 1 M :

(A) $r_1 = r_2 = r_3$ (B) $r_1 > r_2 > r_3$ (C) $r_1 < r_2 < r_3$ (D) All of these

68. **(C)** $r_1 = K[1]^1$

$r_2 = K[1]^2$

$r_3 = K[1]^3$

If $[A] > 1$;

$r_3 > r_2 > r_1$

69. In a second order reaction, first order in each reactant A and B, which one of the following reactant mixtures will provide the highest initial reate :

(A) 0.1 mole of A and 0.1 mole of B in 0.1 litre solution

(B) 0.2 mole of A and 0.2 mole of B in 0.1 litre solution

(C) 0.1 mole of A and 0.1 mole of B in 0.1 litre solution

(D) 0.1 mole of A and 0.1 mole of B is 0.2 litre solution

69. **(B)** rate $= K[A]^1[B]^1 = k\left[\frac{[0.2]}{0.1}\right]\left[\frac{[0.2]}{0.1}\right]$ is maximum.

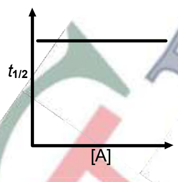
70. Two substances A and B are present such that $[A] = 4\,[B]$ and half life of A is 5 minute and of B is 15 minute. If start decaying at the same time following first order, how much time later will the concentration of both of them would be same ?

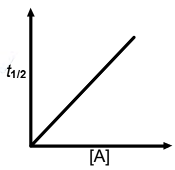
(A) 15 minute (B) 9 minute (C) 6 minute (D) 12 minute

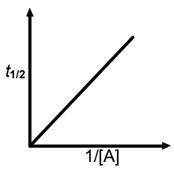
70. **(A)** Amount of A left in n_1 halves $= \frac{[A_0]}{2^{n_1}}$

71. Consider the plots given below, for the types of reaction

$nA \longrightarrow B + C$

(I) 

(II) 

(III) 

The plots respectively correspond to the reaction orders :

(A) 0, 1, 2 (B) 1, 2, 0 (C) 1, 0, 2 (D) 2, 0, 1

71. **(C)**

72. A glucose solution in 100 g of water boils at 100.26° C. If this solution is heated to 101° C. What is the mass of water left at equilibrium ?

(A) 46 g (B) 56 g (C) 26 g (D) 36 g

72. **(C)** $0.26 = K_b \dfrac{n_2}{100} \times 1000$ ...(i)

$l = K_b \dfrac{n_2}{W_1} \times 1000$ ...(ii)

Solving Eqs. (i) and (ii), $W_1 = 26$ g

73. Vapour pressure of an equimolar mixture of benzene and toluene was found to be 80 torr. If the vapour above the liquid phase is condensed in a beaker vapour pressure of this condensate was found to be 100 torr. What is the vapour pressure of pure benzene and pure toluene in the given condition ?

(A) 120 Torr & 40 Torr (B) 100 Torr & 40 Torr

(C) 120 Torr & 80 Torr (D) 40 Torr & 100 Torr

73. **(A)** $\dfrac{1}{2}P_B^0 + \dfrac{1}{2}P_T^0 = 80 \Rightarrow P_B{}^0 + P_T{}^0 = 160$ ...(i)

Mole fractions of benzene and toluene in the vapour phase:

$$x_B = \frac{P_B^0}{160} \text{ and } x_T = \frac{P_T^0}{160}$$

Now, these mole fractions will be the mole fractions of benzene and toluene in the liquid phase of condensate:

$$\Rightarrow \quad \frac{P_B^0}{160} . P_B^0 + \frac{P_T^0}{160} . P_T^0 = 100$$

$$\Rightarrow \quad P_B^0 + P_T^0 = 160 \times 100$$ ...(ii)

Solving, Eqs. (i) and (ii) $P_B{}^0 = 120$ torr and $P_T{}^0 = 40$ torr.

74. If mole fraction of the solvent in a solution decreases, then

(A) Vapour pressure of solution increases

(B) Boiling point decreases

(C) Osmotic pressure increases

(D) Freezing point of solution increases

74. **(C)** Mole fraction of solvent decreases-means mole fraction of solute increases - magnitude of colligative properties increases.

75. At certain temperature, the vapour pressure in torr of methyl alcohol-ethyl alcohol solution is represented by $p = 119x + 135$

where x is the mole fraction of methyl alcohol, then the vapour pressure of pure ethyl alcohol at this temperature is

(A) 135 Torr (B) 16 Torr (C) 0.88 Torr (D) 254 Torr

75. **(A)** $P = 119\,x + 135$

Vapour pressure of pure ethyl alcohol, it means mole fraction of methyl alcohol is 0 (zero) at that temperature. Hence, $P = 0 + 135 = 135$